REKURZIVNI OBRAZCI $\sin(n\alpha)$ in $\cos(n\alpha)$

Razvijem obrazce

n	$S_n = \sin(n\alpha)$	$C_n = \cos(n\alpha)$
1	$\sin\alpha$	$\cos\alpha$
2	$2\sin\alpha\cos\alpha$	$2\cos^2\alpha - 1$
3	$4\sin\alpha\cos^2\alpha - \sin\alpha$	$4\cos^3\alpha - 3\cos\alpha$
4	$8\sin\alpha\cos^3\alpha - 4\sin\alpha\cos\alpha$	$8\cos^4\alpha - 8\cos^2\alpha + 1$
⋮		

Postavim $\sin\alpha = x$ in $\cos\alpha = y$

n	S_n	C_n
1	x	y
2	$2xy$	$2y^2 - 1$
3	$4xy^2 - x = 2y(2xy) - x$	$4y^3 - 3y = 2y(2y^2 - 1) - y$
4	$8xy^3 - 4xy \overset{?}{=} 2y(4xy^2 - x) - 2xy$	$8y^4 - 8y^2 + 1 \overset{?}{=} 2y(4y^3 - 3y) - (2y^2 - 1)$
⋮		

Lahko opazim, da sta S_n in C_n rekurzivno:

$S_1 = x$
$S_2 = 2xy$
$S_3 = 2y \cdot S_2 - S_1$
$S_4 = 2y \cdot S_3 - S_2$
⋮
$S_n = 2y \cdot S_{n-1} - S_{n-2}$

$C_1 = y$
$C_2 = 2y^2 - 1$
$C_3 = 2y \cdot C_2 - C_1$
$C_4 = 2y \cdot C_3 - C_2$
⋮
$C_n = 2y \cdot C_{n-1} - C_{n-2}$

Koliko bo $\sin 5\alpha = ?$

$\sin 5\alpha = 2y\, S_4 - S_3$
$\overset{!}{=} 2\cos\alpha\,(8\sin\alpha\cos^3\alpha - 4\sin\alpha\cos\alpha) - (4\sin\alpha\cos^2\alpha - \sin\alpha)$

$\sin 5\alpha = 16\cos^4\alpha\,\sin\alpha - 12\sin\alpha\cos^2\alpha + \sin\alpha$

by Hana