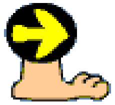
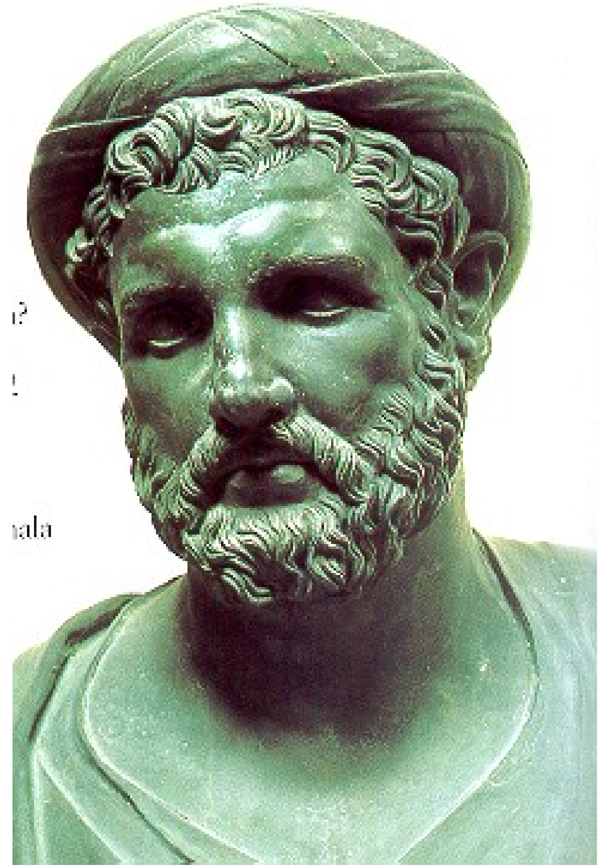
Trigonometrija

v par korakih 

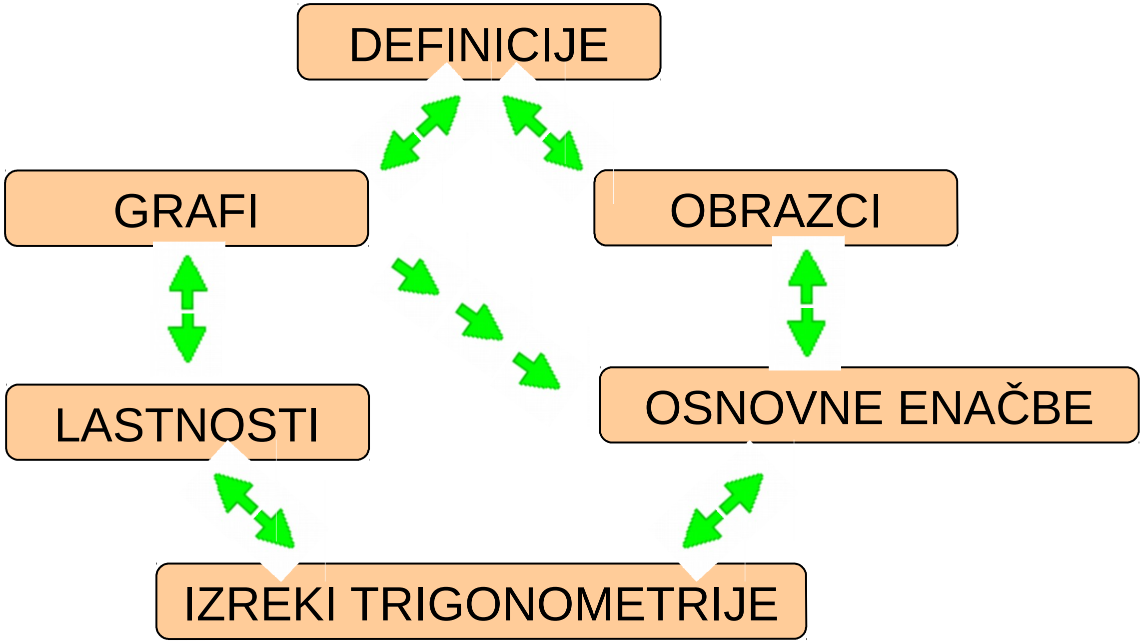
Zahvala gre tudi

očetu trigonometrije, gospodu
Abu Abdullah Muhammad Ibn Musa
al-Khwarizmi (770-840)



In gospodu **Pitagori**
(približno 540-500 pr.Kr.), ki je
kot peteršilj prav povsod.

KAKO VSE TO ZGLEDA?



Trik, da čimprej osvojimo vso to zadevo:

rit na stolici in glava v bukvah!

DEFINICIJE

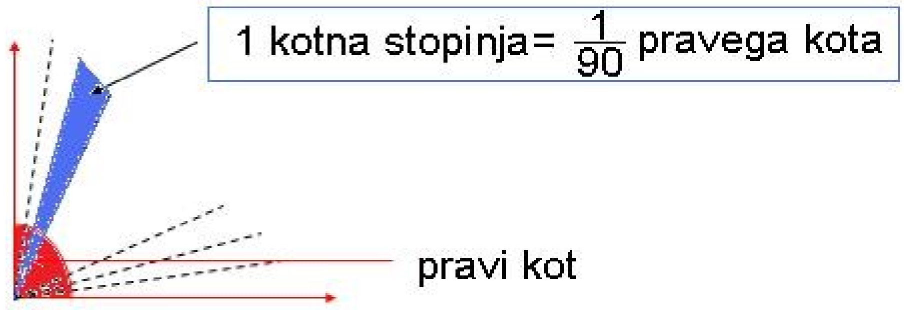
MERSKIH ENOT

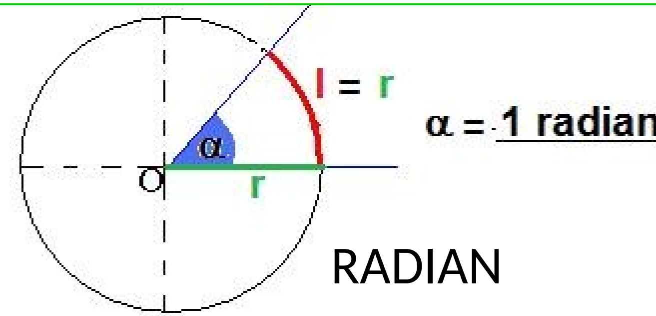
SINUSA

KOSINUSA

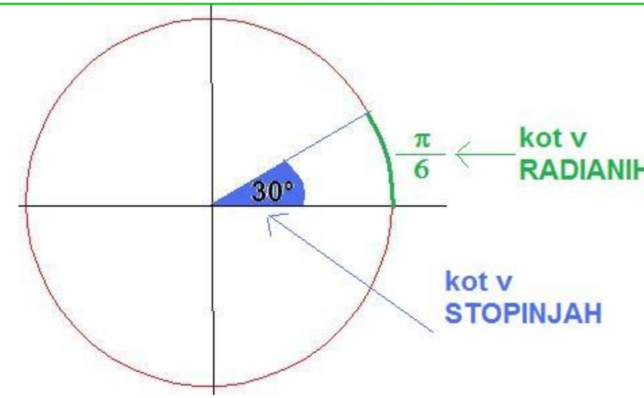
TANGENSA

KOTNA STOPINJA





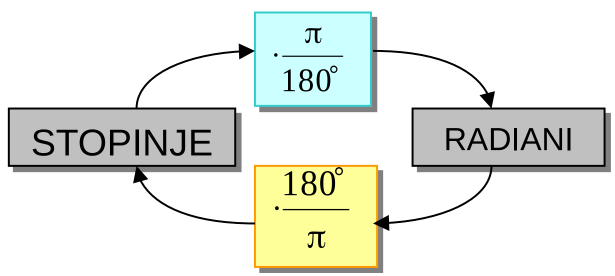
1 radian je središčni kot s polmerom enakim pripadajočemu loku



- **Radini so realna števila, ki predstavljajo dolžino loka pripadajočega središčnega kota.**

IZ ENE MERSKE ENOTE V DRUGO:

$$\alpha^\circ = \frac{180^\circ}{\pi}\alpha$$

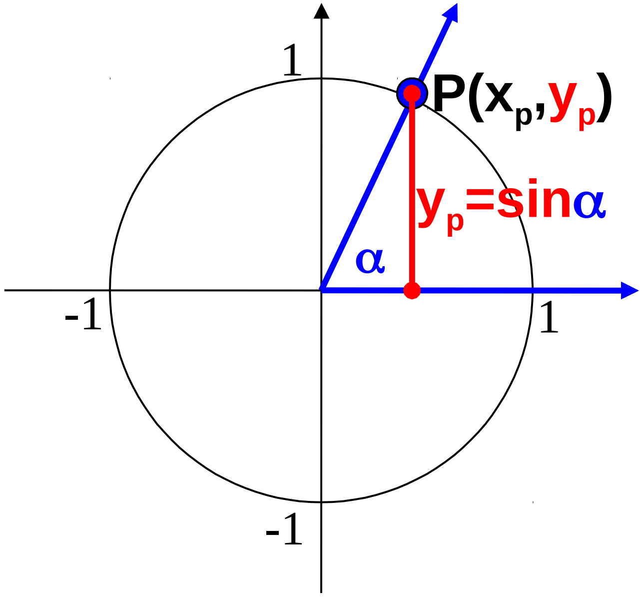


$$\alpha = \frac{\pi}{180^\circ}\alpha^\circ$$

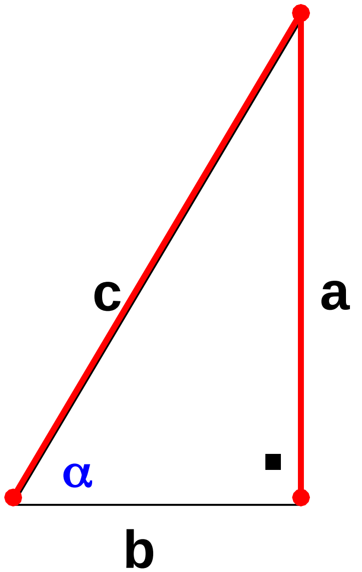
DEFINICIJE

sinα

Sinus kota (sinα) je **ordinata** točke, kjer gibljivi krak kota seče enotsko krožnico.



Sinus kota (sinα) je razmerje med kotu nasprotno kateto in hipotenuzo v pravokotnem trikotniku.

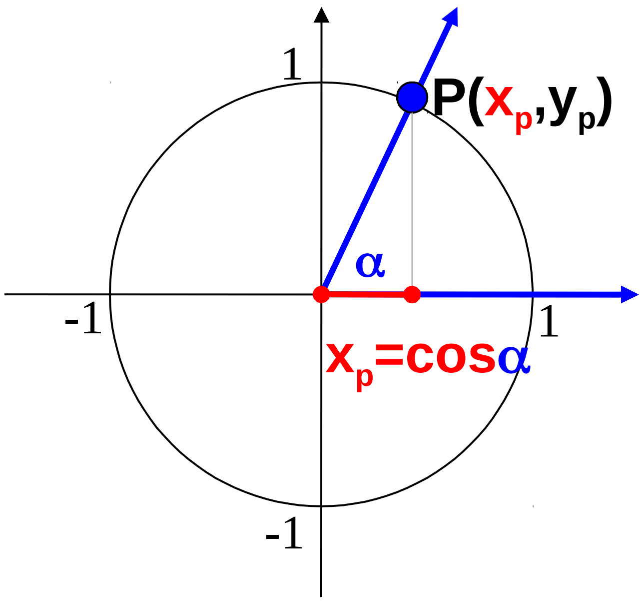


$$\sin\alpha = \frac{a}{c}$$

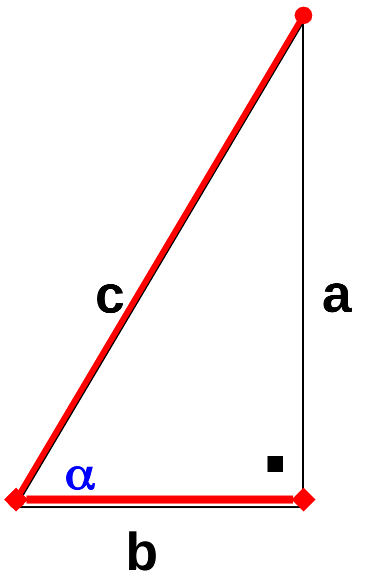
LASTNOSTI | SINUSOIDA | DEFINICIJE

cosα

Kosinus kota (cosα) je **abscisa** točke, kjer gibljivi krak kota seče enotsko krožnico.



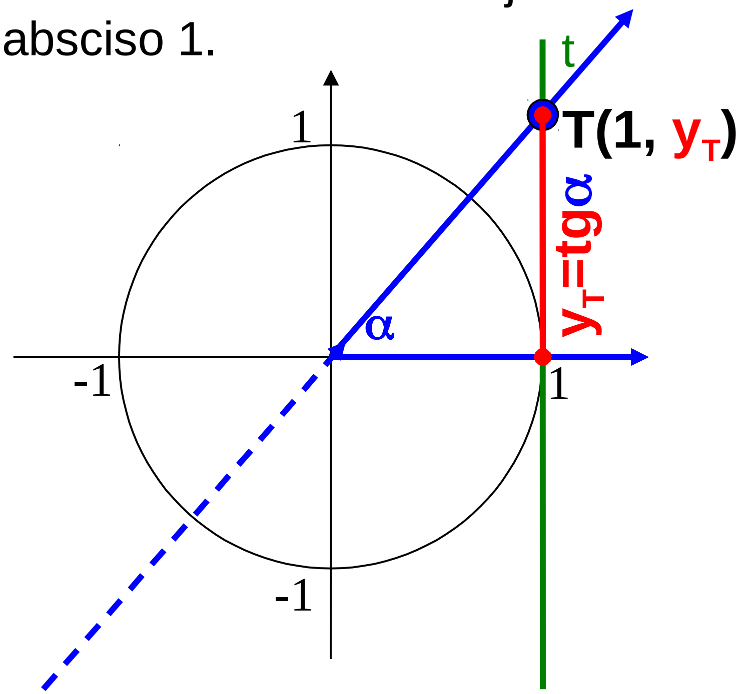
Kosinus kota (cosα) je razmerje med kotu priležno kateto in hipotenuzo v pravokotnem trikotniku.



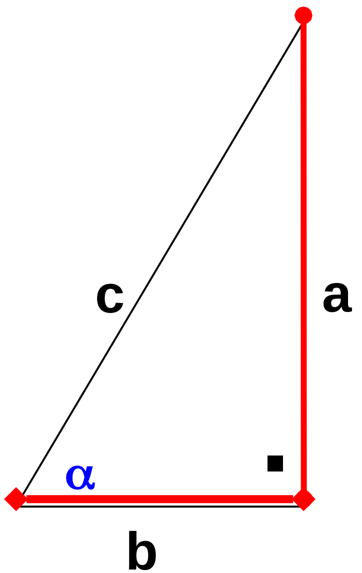
$$\cos\alpha = \frac{b}{c}$$

tgα

Tangens kota (tgα ali tanα) je **ordinata** točke, kjer podaljšek gibljivega kraka kota seče tangento na enotsko krožnico v njen točki z absciso 1.

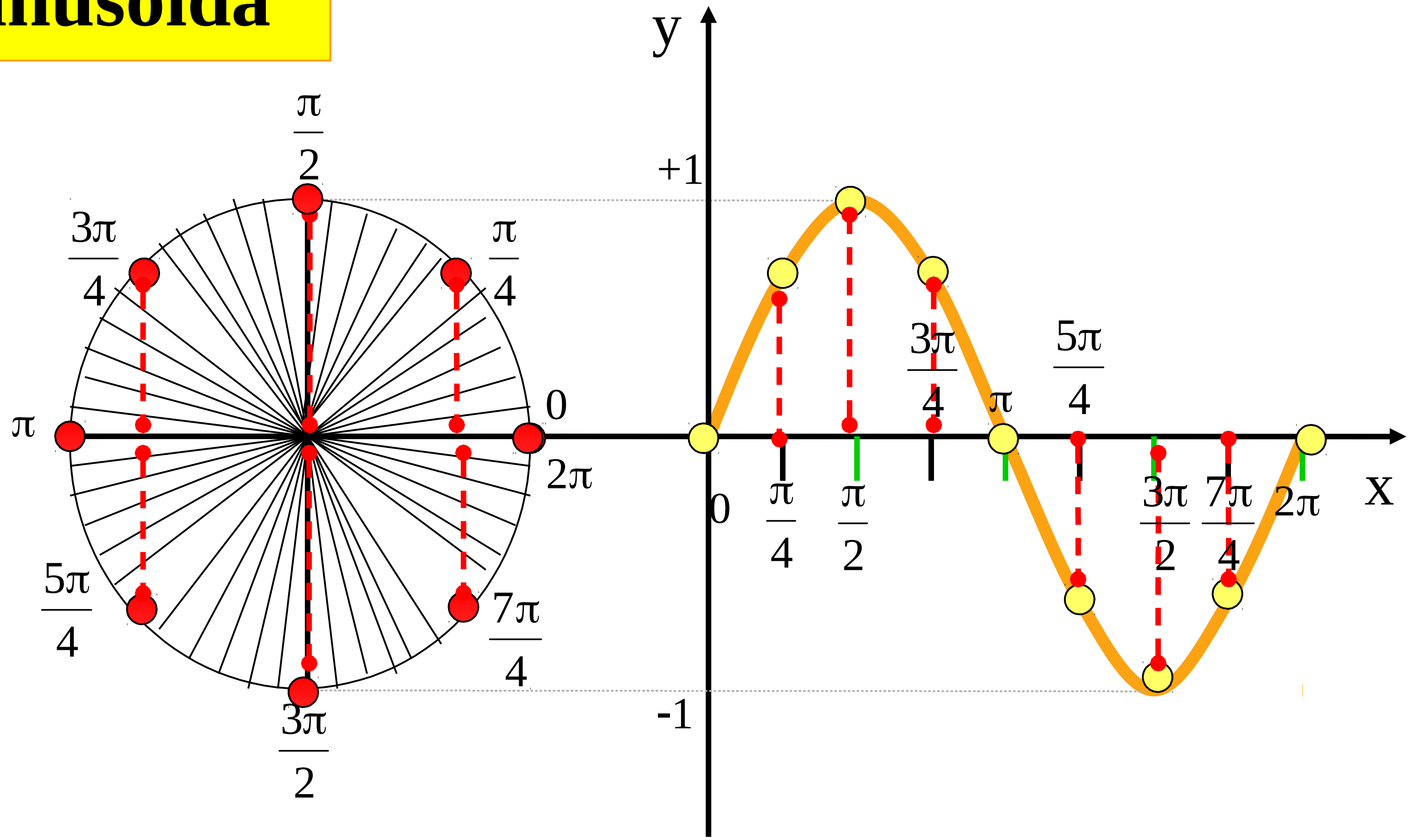


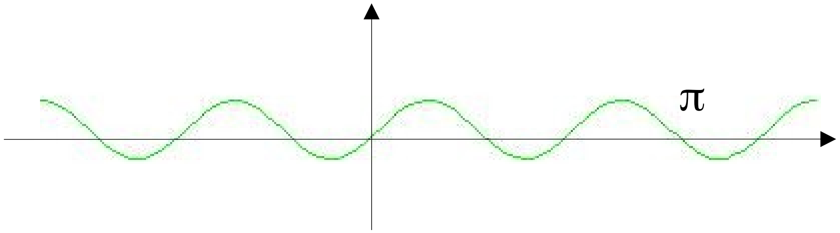
Tangens kota (tga ali tanα) je razmerje med kotu nasprotno kateto in priležno kateto v <u>pravokotnem</u> trikotniku.



$$tg\alpha = \frac{a}{b}$$

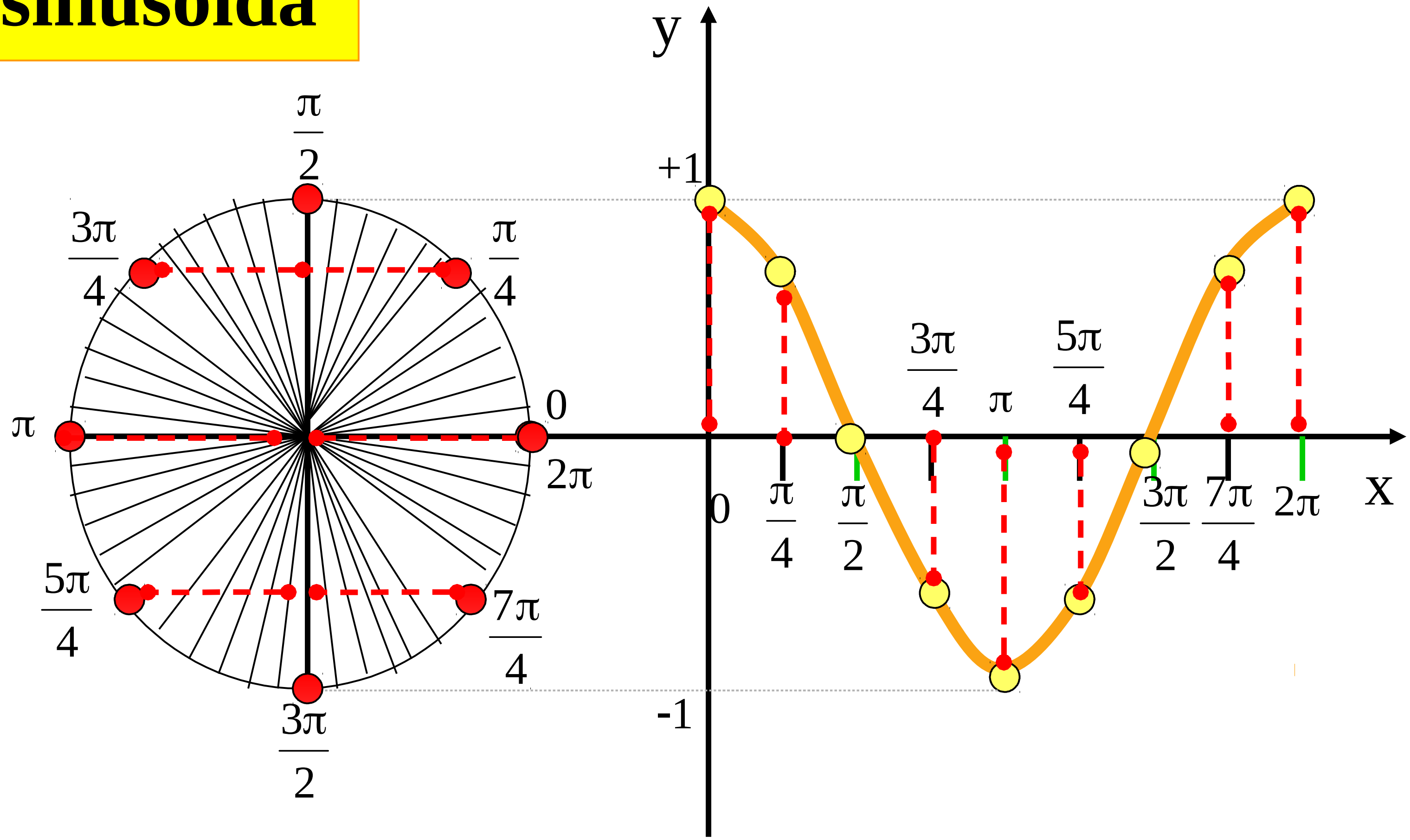
Sinusoida

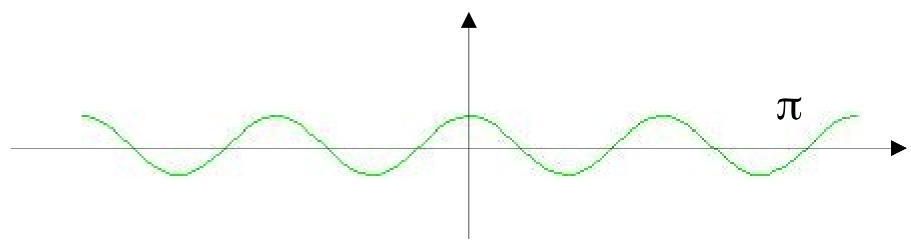




LASTNOSTI DEFINICIJE

Kosinusoida

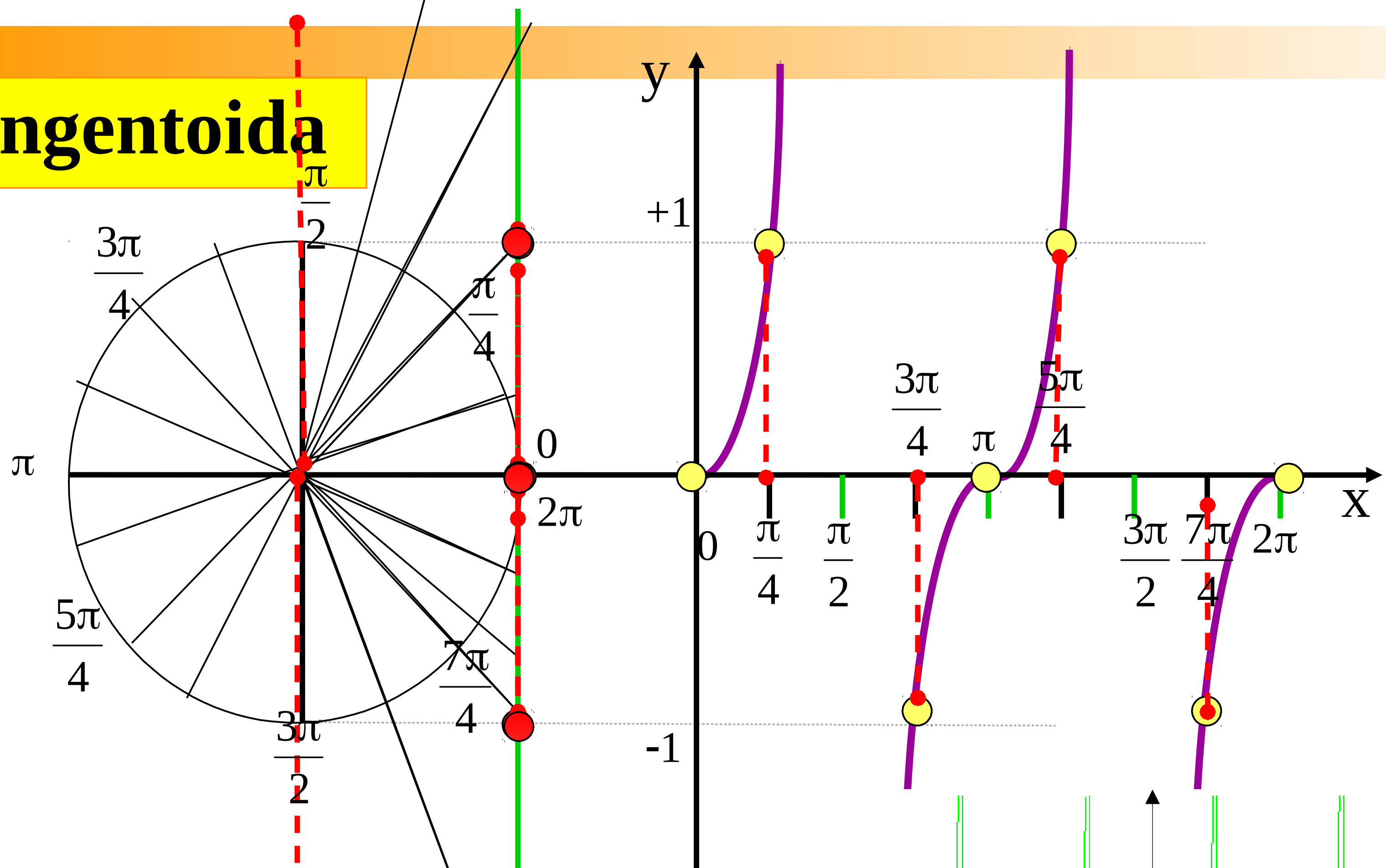


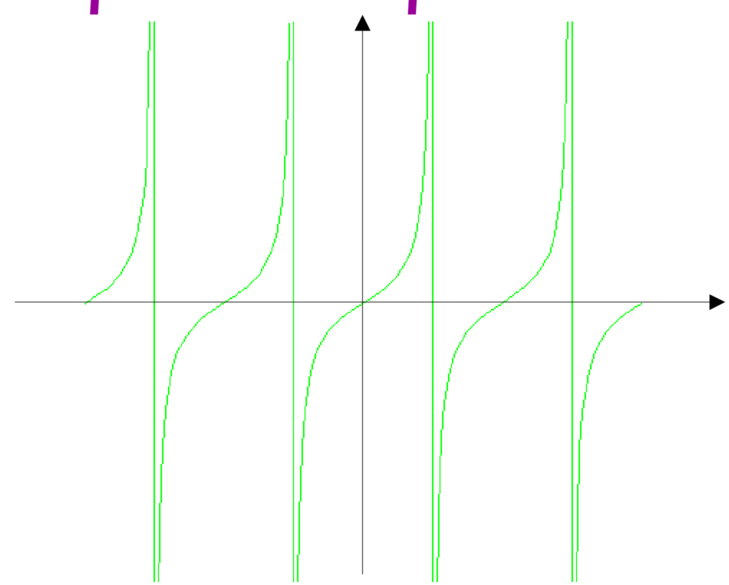


LASTNOSTI

DEFINICIJE

Tangentoida



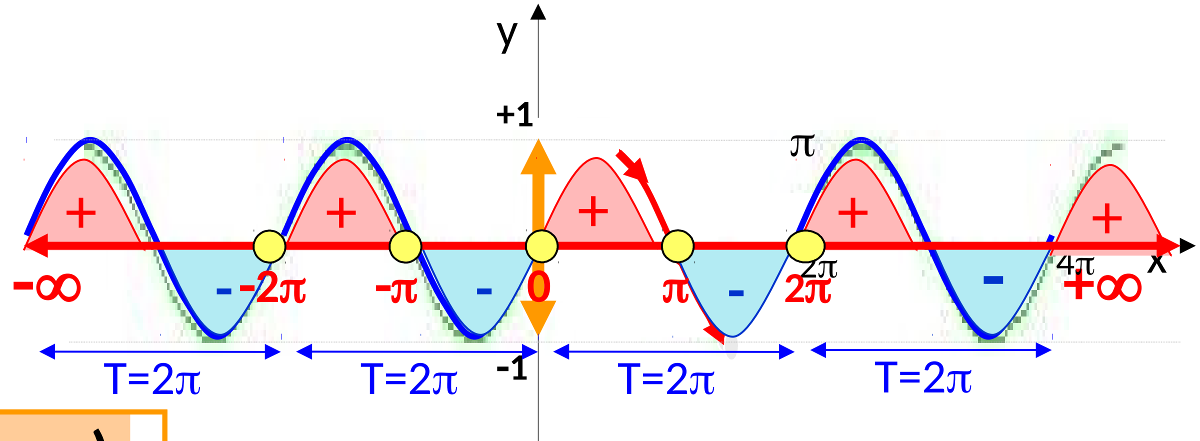


LASTNOSTI

DEFINICIJE

LASTNOSTI

y=sinx

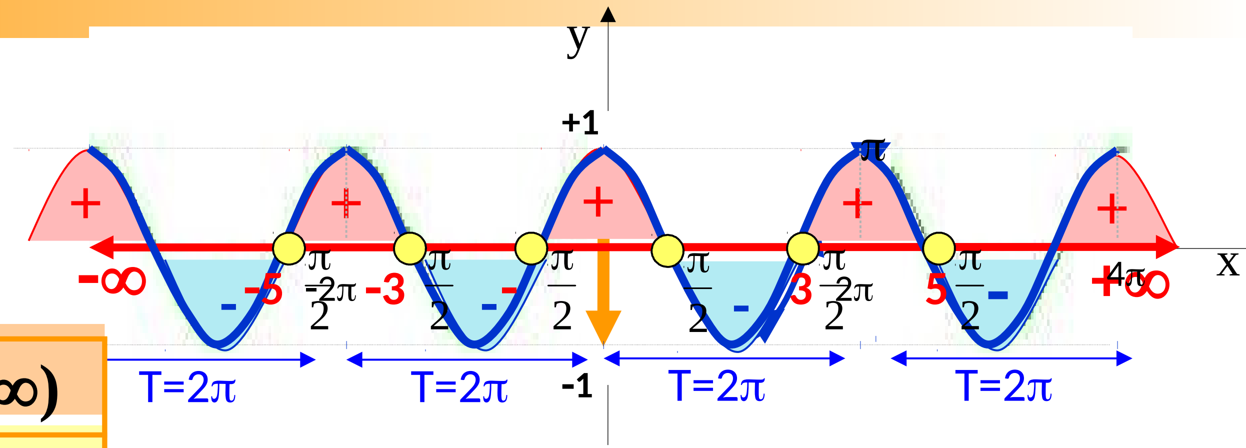


Domena	**(-∞,+∞)**			
Zaloga vrednosti	**[-1,1]**			
Perioda sin (x+2πk)=sinx	**T=2π**			
Ničle	**sin (±kπ)**	**sin0**	**sin(±π)**	**sin (±2π)**
	I.Kvadrant	**II. Kvadrant**	**III. Kvadrant**	**IV. Kvadrant**
Predznaki	**pozitiven**	**pozitiven**	**negativen**	**negativen**
Monotonost	**narašča**	**pada**	**pada**	**narašča**

SINUSOIDA DEFINICIJE

LASTNOSTI

y=cosx



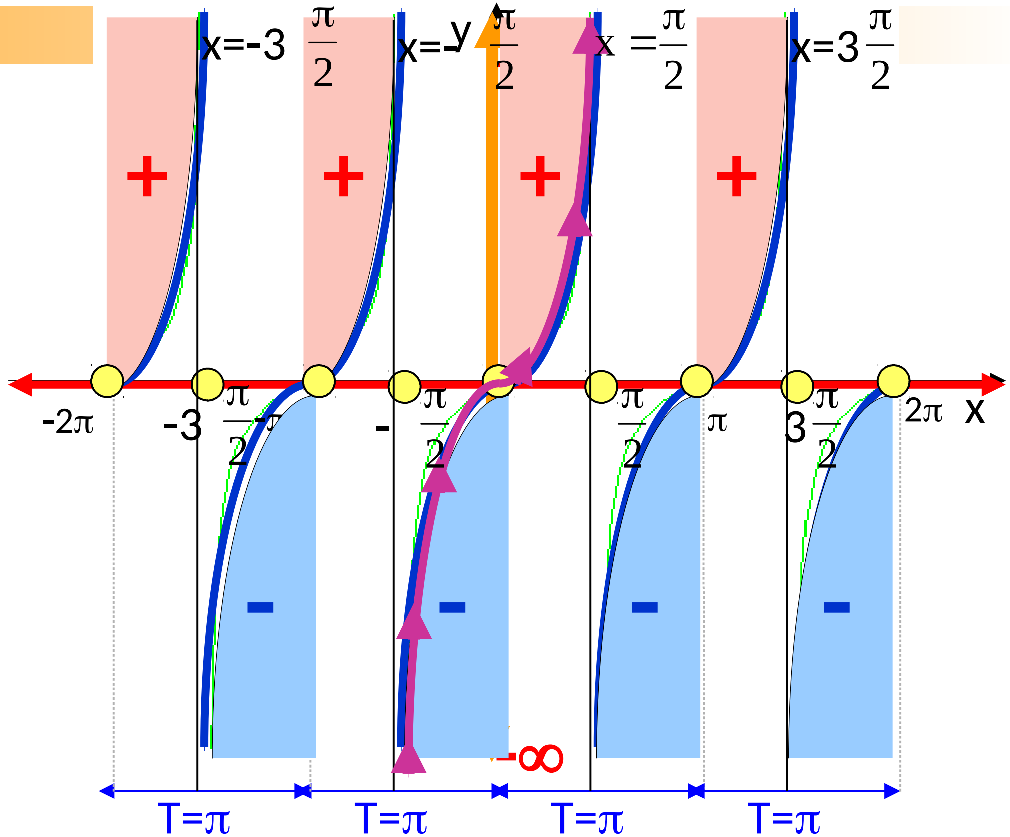
Domena	**(-∞,+∞)**			
Zaloga vrednosti	**[-1,1]**			
Perioda $\cos(x+2\pi k)=\cos x$	$T=2\pi$			
Ničle	$\cos((2k+1)\frac{\pi}{2})$	$\cos(\pm\frac{\pi}{2})$	$\cos(\pm 3\frac{\pi}{2})$	$\cos(\pm 5\frac{\pi}{2})$
	I.Kvadrant	**II. Kvadrant**	**III. Kvadrant**	**IV. Kvadrant**
Predznaki	pozitiven	negativen	negativen	pozitiven
Monotonost	pada	pada	narašča	narašča

KOSINUSOIDA DEFINICIJE

LASTNOSTI

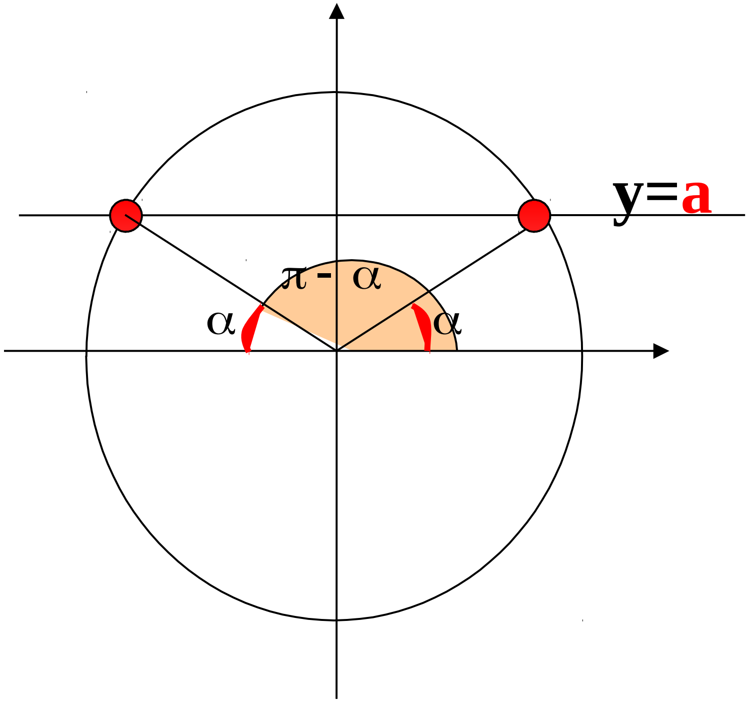
y=tgx

TANGESOIDA | DEFINICIJE

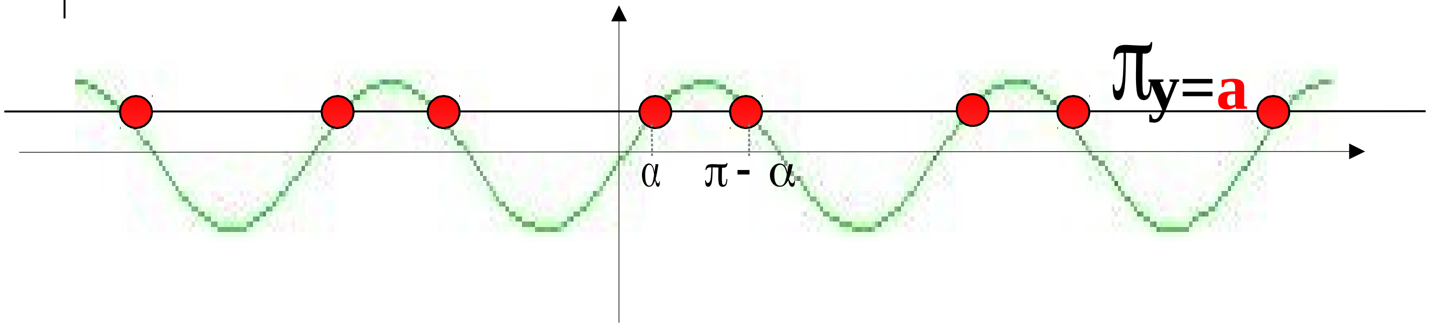


Domena	$\Re\backslash\{x \neq (2k+1)\frac{\pi}{2}\}$			
Zaloga vrednosti	$(-\infty,+\infty)$			
Perioda tg (x+kπ)=tgx	T=π			
Asimptote	$x=(2k+1)\frac{\pi}{2}$	$x=(\pm\frac{\pi}{2})$	$x=(\pm 3\frac{\pi}{2})$	$x=(\pm 5\frac{\pi}{2})$
Ničle	tg (±kπ)	tg0	tg(±π)	tg (±2π)
	I.Kvadrant	II. Kvadrant	III. Kvadrant	IV. Kvadrant
Predznaki	pozitiven	negativen	pozitiven	negativen
Monotonost	narašča	narašča	narašča	narašča

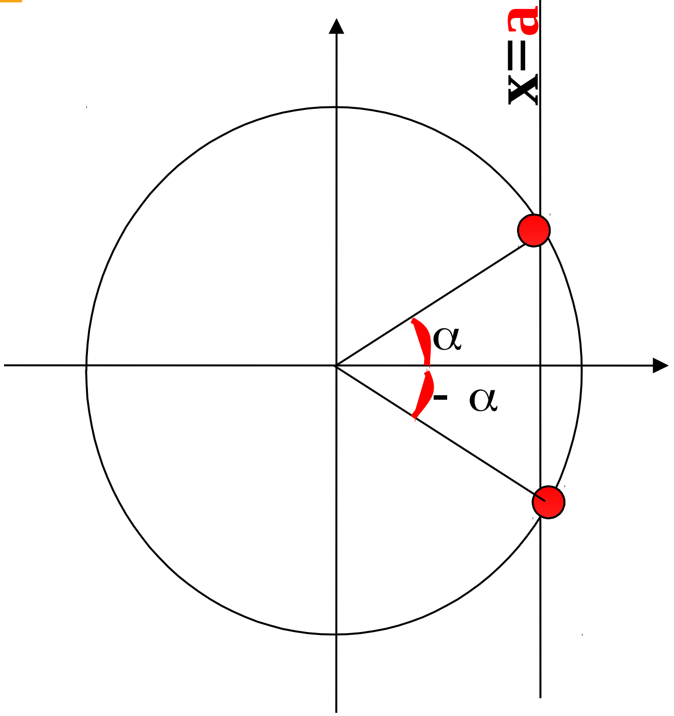
sinα=a



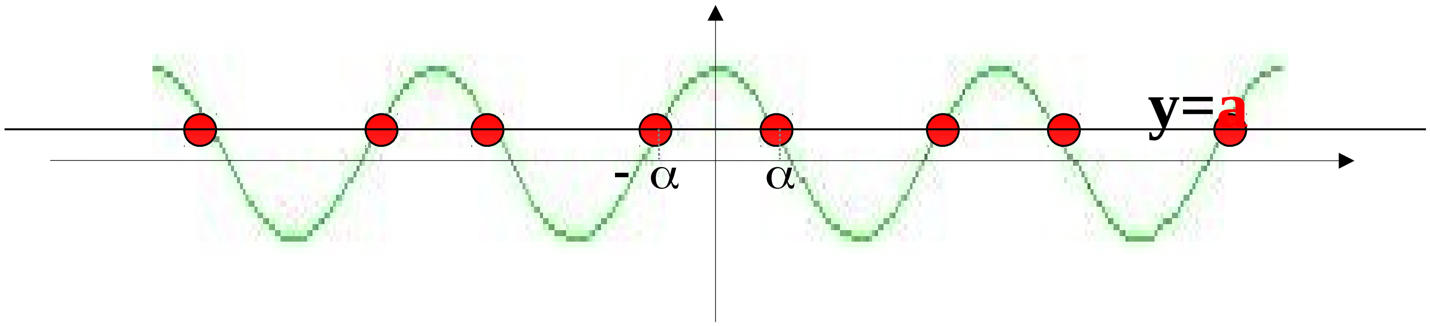
$$\text{rešitve} = \begin{cases} x = \alpha + 2\pi k \\ x = (\pi - \alpha) + 2\pi k \end{cases} \quad \text{kjer } k \in Z$$



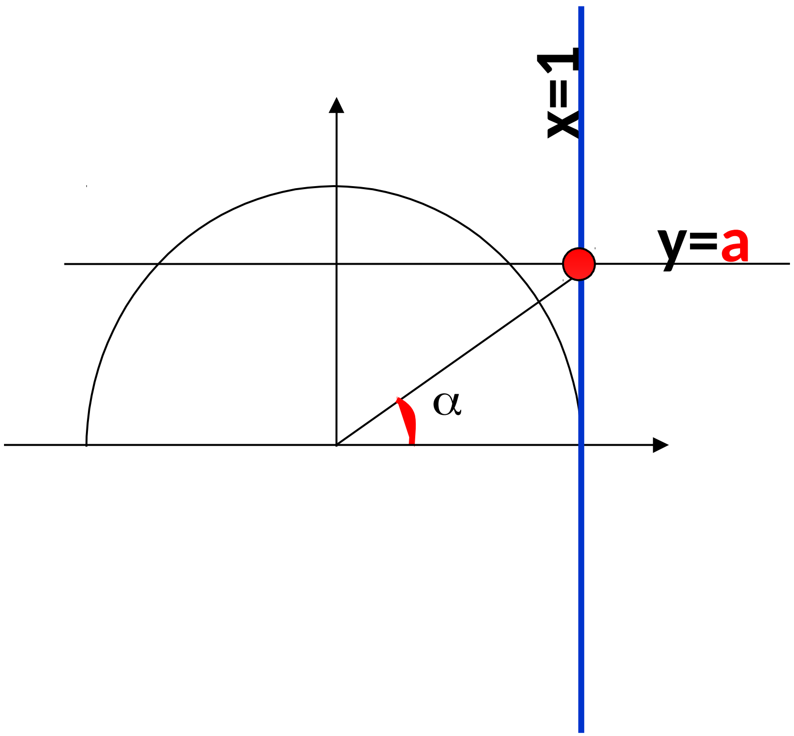
$\cos\alpha = a$



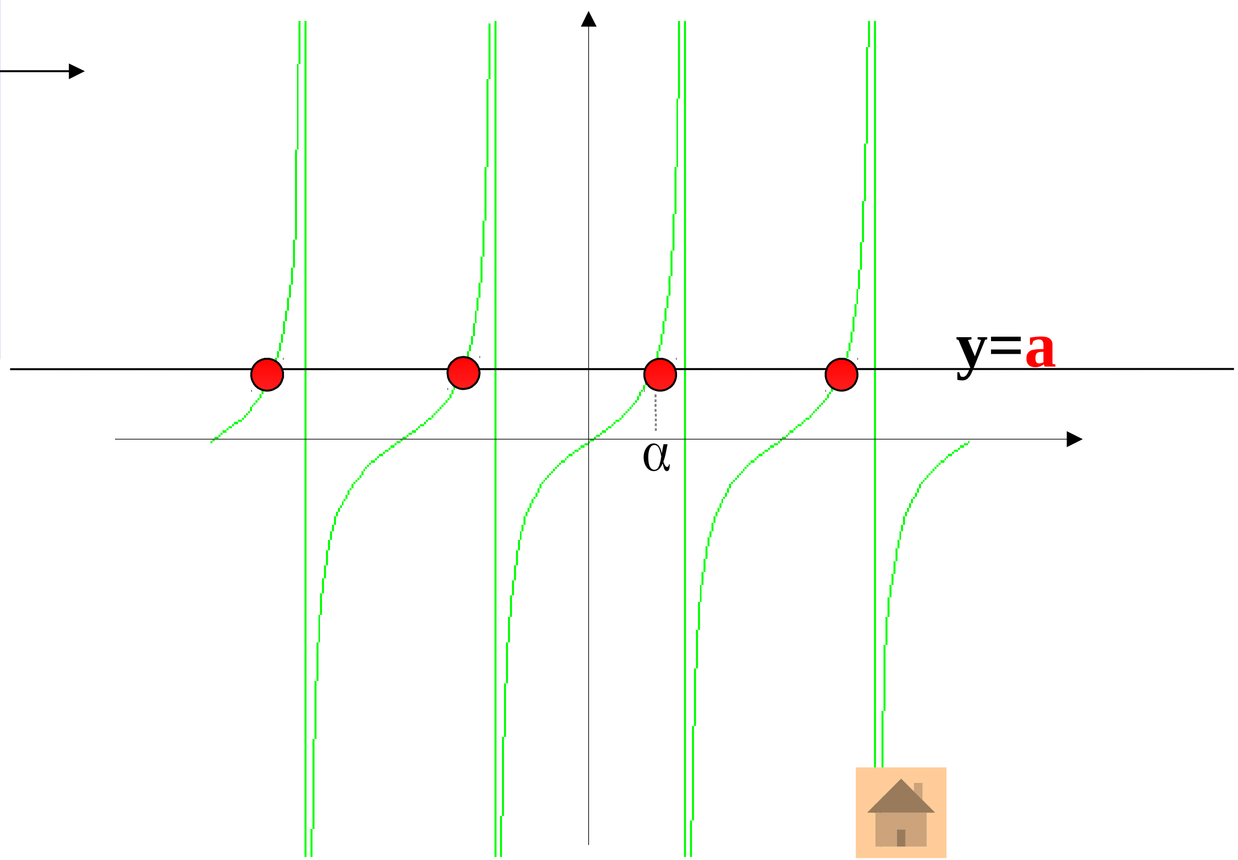
$$\text{rešitve} = \left\langle \begin{array}{l} x = \alpha + 2\pi k \\ x = -\alpha + 2\pi k \end{array} \right. \quad \text{kjer } k \in Z$$



tgα=a



rešitve $\rightarrow$ $x = \alpha + \pi k \quad \text{kjer } k \in Z$



Osnovni zvezi

$$\sin^2 x + \cos^2 x = 1$$

$$\mathrm{tg}x = \frac{\sin x}{\cos x}$$

Vsota in razlika kotov

$$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \sin\beta \cdot \cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$$

$$\mathrm{tg}(\alpha \pm \beta) = \frac{\mathrm{tg}\alpha \pm \mathrm{tg}\beta}{1 \mp \mathrm{tg}\alpha \cdot \mathrm{tg}\beta}$$

Kotne funkcije v osnovnih kotih

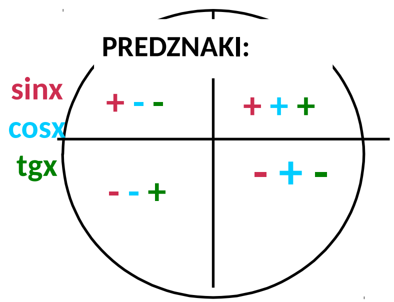
α	α°	$\sin\alpha$	$\cos\alpha$	$\mathrm{tg}\alpha$
0	0	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	∞
π	180°	0	-1	0
$\frac{3\pi}{2}$	270°	-1	0	∞
2π	360°	0	1	0

Polovični kot

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\mathrm{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$



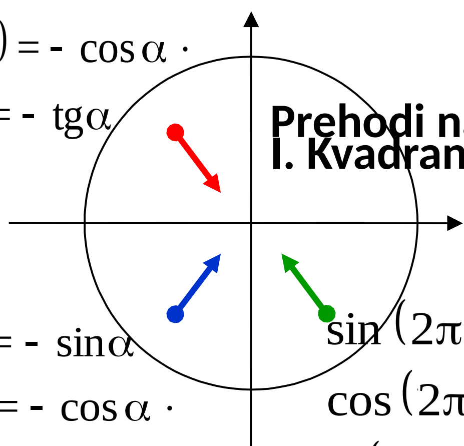
matHELP

$$\sin(\pi - \alpha) = \sin\alpha$$

$$\cos(\pi - \alpha) = -\cos\alpha$$

$$\mathrm{tg}(\pi - \alpha) = -\mathrm{tg}\alpha$$

Prehodi na I. Kvadrant



$$\sin(\pi + \alpha) = -\sin\alpha$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

$$\mathrm{tg}(\pi + \alpha) = \mathrm{tg}\alpha$$

$$\sin(2\pi - \alpha) = -\sin\alpha$$

$$\cos(2\pi - \alpha) = \cos\alpha$$

$$\mathrm{tg}(2\pi - \alpha) = -\mathrm{tg}\alpha$$

Kofunkcije

$$\mathrm{cosec}\,x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\mathrm{ctg}x = \frac{1}{\mathrm{tg}x}$$

Nasprotni koti

$$\sin(-\alpha) = -\sin\alpha$$

$$\cos(-\alpha) = \cos\alpha$$

$$\mathrm{tg}(-\alpha) = -\mathrm{tg}\alpha$$

Dvojni kot

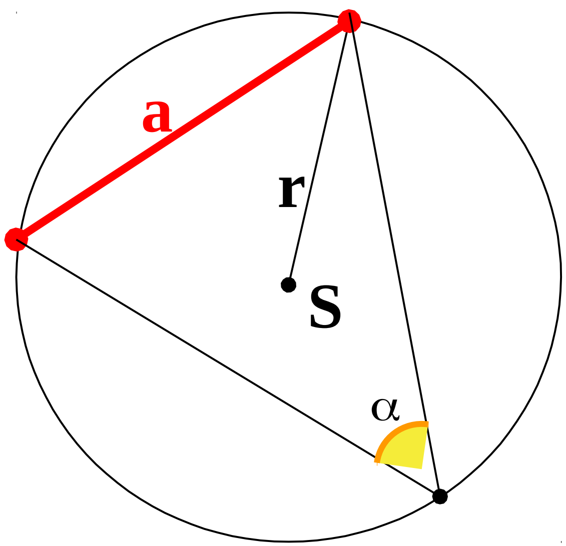
$$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\mathrm{tg}2\alpha = \frac{2\mathrm{tg}\alpha}{1 - \mathrm{tg}^2\alpha}$$

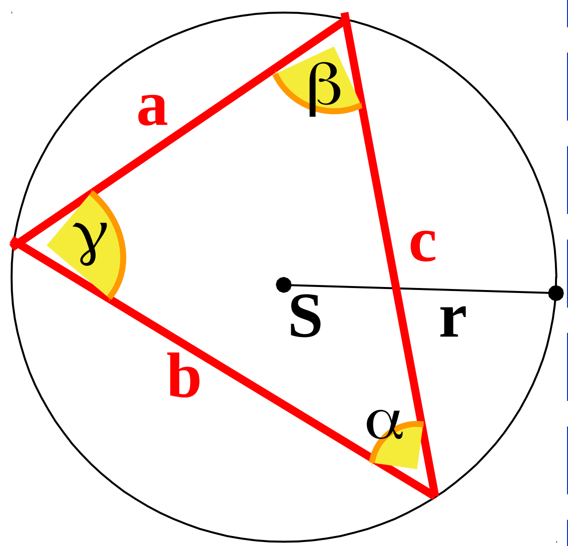
Izreki trigonometrije

Tetivni izrek



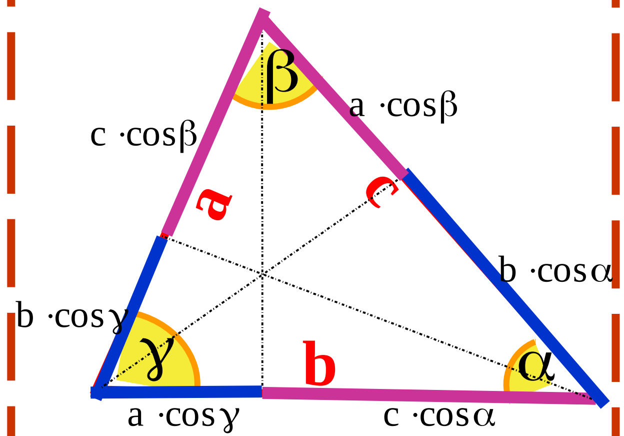
$$a = 2r\sin\alpha$$

Sinusni izrek



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r$$

Projicijski izre



$$b = a\cos\gamma + c\cos\alpha$$

$$a = b\cos\gamma + c\cos\beta$$

$$c = a\cos\beta + b\cos\alpha$$