3. DIAGNOSTIČNI TEST

1. $f(x) = \frac{1}{2x+1}$ $g(x) = \frac{x+1}{4-3x}$

$h = f \circ g$

$f(g(x)) = \frac{1}{2((x+1)/(4-3x))+1} = \frac{1}{(-x+6)/(4-3x)} = \frac{4-3x}{-x+6}$

$h^{-1}(x)$: $x = \frac{4-3y}{-y+6}$

$-xy + 6x = 4 - 3y$

$3y - xy = 4 - 6x$

$y(3-x) = 4-6x$

$y = \frac{4-6x}{3-x}$

2. $h = f \circ g$ $f(x) = \frac{1}{x-1}$ $g(x) = \sqrt{2x-1}$

$f(g(x)) = \frac{1}{\sqrt{2x-1}-1} = \frac{\sqrt{2x-1}+1}{2x-2}$

poli: $2x-2=0$

$2x = 2$

$\underline{\underline{x=1}}$

območje nedefiniranosti:

$2x-1<0$

$2x<1$

$x<\frac{1}{2}$

Definicijsko območje: $[\frac{1}{2}, 1) \cup (1, \infty)$

3. a) $\lim\limits_{x\to 0} \frac{\sin(5x)}{x\cos(3x)} = \lim\limits_{x\to 0} \frac{5\cdot\sin(5x)}{5x\cos(3x)} = \lim\limits_{x\to 0} \frac{5}{\cos(3x)} = \frac{5}{1} = 5$

b) $\lim\limits_{x\to 2} \frac{x^2-3x+2}{1-\sqrt{3x-5}} = \lim\limits_{x\to 2} \frac{(x^2-3x+2)(1+\sqrt{3x-5})}{6-3x} = \lim\limits_{x\to 2} \frac{(x-2)(x-1)(1+\sqrt{3x-5})}{-3(-2+x)} =$

$= \lim\limits_{x\to 2} \frac{(x-1)(1+\sqrt{3x-5})}{-3} = \frac{(2-1)(1+\sqrt{6-5})}{-3} = \frac{1\cdot 2}{-3} = -2/3$

4. a) $\lim\limits_{n\to\infty} \frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n+1}-\sqrt{n}} = \lim\limits_{n\to\infty} \frac{(\sqrt{n}-\sqrt{n-1})\cdot(\sqrt{n}+\sqrt{n-1})\cdot(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}-\sqrt{n})\cdot(\sqrt{n+1}+\sqrt{n})\cdot(\sqrt{n}+\sqrt{n-1})} =$

$= \lim\limits_{n\to\infty} \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n}+\sqrt{n-1}} = \lim\limits_{n\to\infty} \frac{\sqrt{n\cdot(1+\frac{1}{n})}+\sqrt{n}}{\sqrt{n}+\sqrt{n\cdot(1-\frac{1}{n})}} = \lim\limits_{n\to\infty} \frac{\sqrt{n}(\sqrt{1+\frac{1}{n}}+1)}{\sqrt{n}(1+\sqrt{1-\frac{1}{n}})} = \frac{\sqrt{1+0}+1}{1+\sqrt{1-0}} = 1$

b) $\lim\limits_{n\to\infty} \frac{3^{2n-1}+4^{-n}}{9^{n+1}+2^{-5n}+1} = \lim\limits_{n\to\infty} \frac{3^{2n-1}+2^{-2n}}{3^{2n+2}+2^{-5n}+1} = \lim\limits_{n\to\infty} \frac{\frac{3^{2n-1}}{3^{2n}}+\frac{2^{-2n}}{3^{2n}}}{\frac{3^{2n-2}}{3^{2n}}+\frac{2^{-5n}}{3^{2n}}+\frac{1}{3^{2n}}} =$

$= \lim\limits_{n\to\infty} \frac{3^{-1}+\frac{2^{-2n}}{3^{2n}}}{3^{-2}+\frac{2^{-5n}}{3^{2n}}+\frac{1}{3^{2n}}} = \frac{3^{-1}+0}{3^{-2}+0+0} = \frac{9}{3} = 3$

5. $f(x) = \frac{x^2+3x}{x-6}$

$x+2y-7=0$

$y = -\frac{1}{2}x + \frac{7}{2}$

$f'(x) = \frac{(2x+3)(x-6)-(x^2+3x)}{(x-6)^2} = \frac{2x^2-12x+3x-18-x^2-3x}{(x-6)^2} = \frac{x^2-12x-18}{(x-6)^2} = -\frac{1}{2}$

$-\frac{1}{2} = \frac{x^2-12x-18}{(x-6)^2}$

$-\frac{1}{2}(x-6)^2 = x^2-12x-18$

$-\frac{x^2}{2}+6x-18 = x^2-12x-18$

$-\frac{3x^2}{2}+18x=0$

$-x^2+12x=0$

$-x(x-12)=0$

$x_1=0 \quad x_2=12 \quad y_1=0 \quad y_2=30$

$(y-y_1)=k(x-x_1)$

$y=-\frac{1}{2}x$

$(y-y_2)=k(x-x_2)$

$y=\frac{1}{2}x+6+30$

$y=\frac{1}{2}x+36$

6. $f(x) = \frac{1}{4} x^4 - 2x^3 + 4x^2 - 1 \qquad [-1, 4]$

$f'(x) = x^3 - 6x^2 + 8x$

$0 = x^3 - 6x^2 + 8x$

$0 = x(x^2 - 6x + 8)$

$0 = x(x-4)(x-2)$

$x_1 = 0 \quad x_2 = 2 \quad x_3 = 4$

$\underline{\underline{y_1 = -1}} \quad y_2 = 3 \quad \underline{\underline{y_3 = -1}}$

$f(-1) = \frac{1}{4}(-1)^4 - 2(-1)^3 + 4(-1)^2 - 1 = \underline{\underline{5\frac{1}{4}}}$

maximum: $5\frac{1}{4}$ minimum: -1

7. $f(x) = \sqrt{x^2 + 3x - 1} \qquad g(x) = \sqrt{x^2 + x + 3}$

$\sqrt{x^2 + 3x - 1} = \sqrt{x^2 + x + 3}$

$x^2 + 3x - 1 = x^2 + x + 3$

$x = 2$

$f(2) = \sqrt{4 + 6 - 1} = 3 \qquad T(2, 3)$

$f'(2) = \frac{1}{2}(2^2 + 3 \cdot 2 - 1)^{0,5} \cdot (2 \cdot 2 + 3) = \frac{7}{6} = k_1$

$g'(2) = \frac{1}{2}(2^2 + 2 + 3)^{0,5} \cdot (2 \cdot 2 + 1) = \frac{5}{6} = k_2$

$\alpha = \arctan\left|\frac{k_1 - k_2}{1 + k_1 \cdot k_2}\right|$

$\alpha = \arctan \frac{12}{71}$

$\alpha = 9,59$

8.
$$a_n = \frac{1+3+5+7 \dots + (2n-1)}{n^2+1}$$

$$a_n = \frac{n(2\cdot 1 + (n-1)\cdot 2)}{2n^2+2} = \frac{n(2+2n-2)}{2(n^2+1)} = \frac{2n^2}{2(n^2+1)} =$$

$$a_n = \frac{n^2}{n^2+1}$$

$$a_{n+1} - a_n = \frac{(n+1)^2}{(n+1)^2+1} - \frac{n^2}{n^2+1} = \frac{n^2+2n+1}{n^2+2n+2} - \frac{n^2}{n^2+1} =$$

$$= \frac{(n^2+2n+1)(n^2+1) - n^2(n^2+2n+2)}{(n^2+2n+2)(n^2+1)} = \frac{(n^4+2n^3+2n^2+2n+1) - (n^4+2n^3+2n^2)}{(n^2+2n+2)(n^2+1)} =$$

$$= \frac{2n+1}{(n^2+2n+2)(n^2+1)} > 0 \Rightarrow \text{naraščujoče}$$

$$\lim_{n\to\infty} \frac{n^2}{n^2+1} = \lim_{n\to\infty} \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}} = \frac{1}{1+0} = 1 \Rightarrow \text{konvergentno}$$

9. $a_n = 81\sqrt{2}$

$a_{n-1} = 27\sqrt{6}$

$S_n = 121\sqrt{2} + 40\sqrt{6}$

$k = \frac{a_n}{a_{n-1}} = \sqrt{3}$

$a_n = a_1 \cdot k^{n-1} \Rightarrow a_1 = \frac{a_n}{k^{n-1}}$

$$S_n = \frac{a_1\cdot(k^n-1)}{k-1}$$

$$S_n\cdot(k-1) = a_1\cdot(k^n-1)$$

$$S_n\cdot(k-1) = \frac{a_n}{k^{n-1}}\cdot(k^n-1)$$

$$S_n\cdot(k-1) = \frac{a_n\cdot k^n}{k^{n-1}} - \frac{a_n}{k^{n-1}}$$

$$S_n\cdot(k-1) - (a_n\cdot k) = -\frac{a_n}{k^{n-1}}$$

$$\frac{-a_n}{S_n\cdot(k-1) - (a_n\cdot k)} = k^{n-1}$$

NADALJEVANJE NA DRUGI STRANI →

$$\frac{-81\sqrt{2}}{(121\sqrt{2}+40\sqrt{6})(\sqrt{3}-1)-(81\sqrt{2}\cdot\sqrt{3})}=\sqrt{3}^{\,n-1}$$

$$\frac{-81\sqrt{2}}{121\sqrt{6}+120\sqrt{2}-121\sqrt{2}-40\sqrt{6}-81\sqrt{6}}=3^{\frac{n-1}{2}}$$

$$\frac{-81\sqrt{2}}{-\sqrt{2}}=3^{\frac{n-1}{2}}$$

$$\log_3 81=\frac{n-1}{2}$$

$$4=\frac{n-1}{2}$$

$$9=n$$

$$a_1=\frac{a_n}{k^{n-1}}=\sqrt{2}$$

$$a_2=a_1\cdot k=\sqrt{6}$$

$$a_3=a_2\cdot k=3\sqrt{2}$$

10.

$$\log_2(1+x),\ 1+\log_2(3x-1),\ \log_2(3x+5)$$

$$2a_2=a_1+a_3$$

$$2(1+\log_2(3x-1))=\log_2(1+x)+\log_2(3x+5)$$

$$2+2\log_2(3x-1)=\log_2((1+x)(3x+5))$$

$$\log_2 4+\log_2(3x-1)^2=\log_2(3x+5+3x^2+5x)$$

$$\log_2(4(9x^2-6x+1))=\log_2(3x^2+8x+5)$$

$$4(9x^2-6x+1)=3x^2+8x+5$$

$$33x^2-32x-1=0$$

$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}=-\frac{1}{33}$ // $\Rightarrow$ ker: $3x_1-1<0$

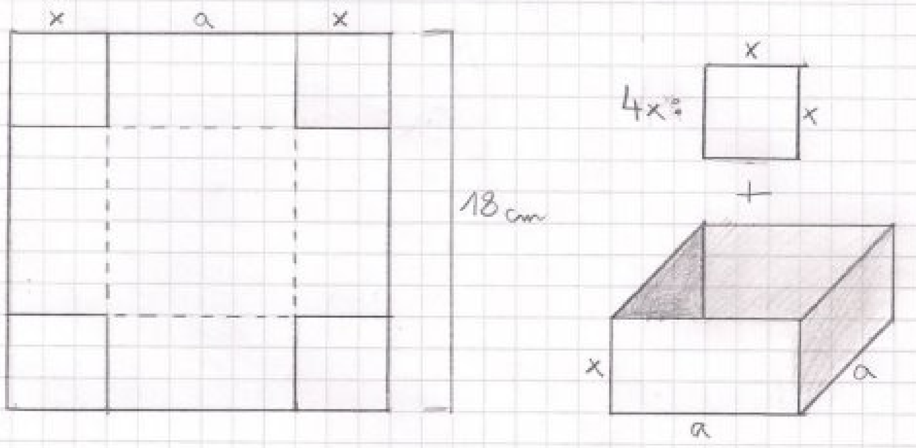
$x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}=1$ ✓

$$\log_2(1+x_2)=1=a_1$$

$$1+\log_2(3x-x_2)=2=a_2$$

$$\log_2(3x+5)=3=a_3$$

11.



$V = a \cdot a \cdot x$ $\qquad x = \frac{18-a}{2}$

$V = a^2 \cdot \frac{18-a}{2}$

$V = \frac{18a^2 - a^3}{2}$

$V' = \frac{(36a - 3a^2)\cdot 2}{2^2} = 0$

$36a - 3a^2 = 0$

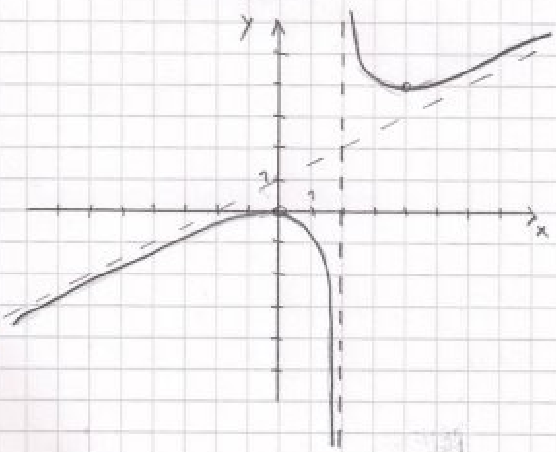
$3a(12 - a) = 0$

$a_1 = 0\,cm$ // => stranica ne mora imeti 0 cm

$a_2 = 12\,cm$ ✓

$x = \frac{18-a}{2} = 3\,cm$

12. $f(x) = \frac{x^2}{2x-4}$



ničle: $x^2 = 0$ $\quad x_{1,2} = 0$

pol: $2x - 4 = 0$ $\quad x = 2$

$f(0)$: $\frac{0}{0-4} = 0$

asimptota: $x^2 : (2x-4) = \frac{x}{2} + 1 = y$

$\frac{x - 2x}{2x}$

ekstremi: $(2x)(2x-4) - (x^2)(2) = 0$

$2x^2 - 8x = 0$ $\qquad x_1 = 0 \quad y_1 = 0 \quad T_1(0,0)$

$2x(x-4) = 0$ $\qquad x_2 = 4 \quad y_2 = 4 \quad T_2(4,4)$