KOTNE FUNKCIJE VAJE

List1

1) Dana je funkcija $f(x)=2\sin x+1$. Določi ničle dane funkcije in nariši graf $g(x)=|f(x)|$.

2) Dana je funkcija $f(x)=2\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$. Nariši graf funkcije in zapiši koordinate ekstremov. Izračunaj $f(2\alpha)$, če je $tg\,\alpha=\frac{1}{4}$. Rezultat naj bo točen.

3) Poenostavi izraz $\sin(\pi+x)+\sin(\pi-x)+\sin\left(x-\frac{\pi}{2}\right)+\sin\left(x+\frac{\pi}{2}\right)$.
$\{R:0\}$

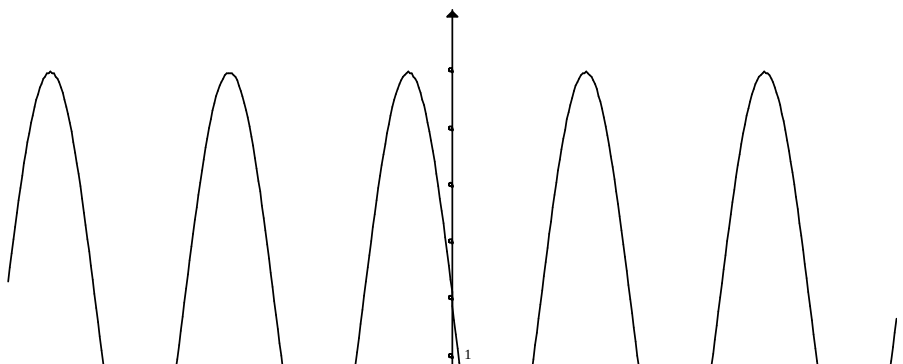
4) Poenostavi izraz $\dfrac{\sin\left(\frac{\pi}{2}-x\right)-\cos^3 x}{\sin\left(2x+\frac{\pi}{4}\right)}$, če je $x\neq\frac{k\pi}{2}$, $k\in Z$.
$\left\{R:\frac{\sin x}{2}\right\}$

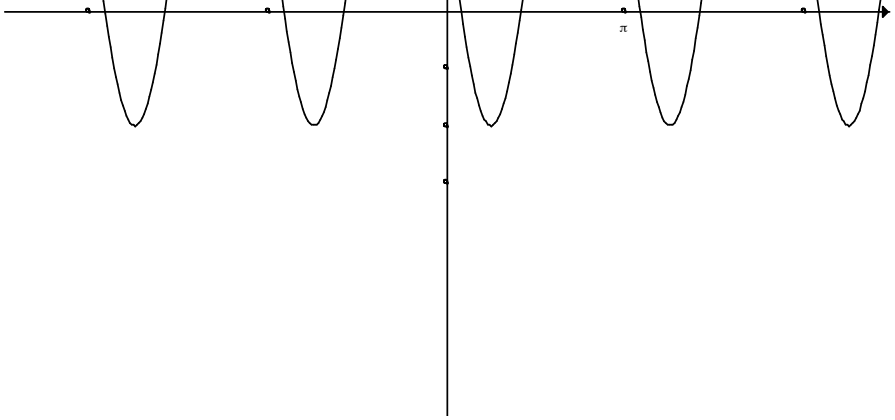
5) Naj bo α topi kot $\left(\frac{\pi}{2}<\alpha<\pi\right)$ in $\sin\alpha=\frac{3}{5}$. Izračunaj točni vrednosti $\sin 2\alpha$ in $tg\left(\alpha+\frac{\pi}{4}\right)$.
$\left\{R:\sin 2\alpha=-\frac{24}{25}, tg\left(\alpha+\frac{\pi}{4}\right)=\frac{1}{7}\right\}$

6) Pokaži, da velja $\frac{1}{2}-\frac{1}{2}\sin\left(2x+\frac{\pi}{2}\right)=\sin^2 x$ za vsak x.

7) Pokaži, da velja $\frac{\sin x\,(\cos 2x+1)}{\cos x\sin 2x}=1$ za vsak $x\neq\frac{k\pi}{2}$, $k\in Z$.

8) Na sliki je graf funkcije $f(x)=A\sin(\omega x+\phi)+B$. Zapiši funkcijski predpis in določi periodo.





List2

1. Natančno izračunaj $\sin 15^\circ$ in $\cos 105^\circ$.
2. Izračunaj $\cos(\frac{5\pi}{3})\cos(\frac{11\pi}{6}) - \sin(\frac{5\pi}{3})\sin(\frac{11\pi}{6})$
3. Izračunaj $\sin(\frac{\pi}{3} - \alpha)$, če je $0 < \alpha < \pi$ in $\cos\alpha = -\frac{3}{5}$.
4. Izračunaj $\sin\alpha$, če je $\sin(\frac{\pi}{4} - \alpha) = -\frac{2}{3}$ in $\frac{\pi}{4} < \alpha < \frac{5\pi}{4}$.
5. Sinusa dveh ostrih kotov sta $\frac{1}{\sqrt{5}}$ in $\frac{1}{\sqrt{10}}$. Koliko je vsota teh dveh kotov?
6. Izračunaj $\cos(\alpha - \beta + \gamma)$, če je $\cos\alpha = \frac{12}{13}$, $\sin\beta = \frac{8}{17}$, $\sin\gamma = \frac{3}{5}$ in so vsi trije koti ostri.
7. Izračunaj $\cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} + \cos^2\frac{9\pi}{8}$.
8. Zapiši kot produkt $\sqrt{2} + 2\cos\alpha$.
9. Zapiši kot produkt $1 + \sin x - \cos x$.
10. Razčleni $2\sin^2 x \sin 2x$.
11. Zapiši nihanje $\sqrt{3}\sin x - \cos x$ v obliki $A\sin(\omega x + \varphi)$.
12. Določi osnovno periodo za funkcijo $2\cos^2 x + \sin^2 x$ (zapiši jo na isti način kot zgoraj).
13. Naj bo $ctg\alpha = \sqrt{2}$. Izračunaj vrednosti preostalih kotnih funkcij.
14. Izračunaj $tg75^\circ$.
15. Naj bo $\pi < \alpha < 2\pi$ in $tg\alpha = \frac{4}{3}$. Izračunaj $tg2\alpha$ in $\cos 2\alpha$.
16. Izračunaj $arctg(2\sin\frac{\pi}{2})$, $\arccos(\cos 10)$, $arctg(tg75)$, $\arcsin(\sin\frac{5\pi}{4})$
17. Reši enačbe:
    a) $ctg(\frac{\pi}{4} - 2x) = \sqrt{3}$
    b) $tg(30^\circ - x) = -2$
    c) $4\cos 3x = 1$
    d) $\sin^2(\frac{2x}{3} - \frac{\pi}{6}) = \frac{3}{4}$
    e) $2tg^2 x + 3\cos^{-1} = 0$
    f) $\sin^2 x + 4\cos x + 4 = 0$
    g) $4\cos^2 x - 3tg^{-2} x + 1 = 0$
    h) $\sin x - \cos x = 0$
    i) $14\sin^2 x + 3\sin 2x = 4$
    j) $2\cos^2 x + \sin 2x = 2$
    k) $\sin 4x - \cos 3x = \cos 2x + \sin x$
    l) $6\sin x \cdot \cos^2 x + 2 = 3\cos x + 2\sin 2x$
    m) $2\sin x + 4tgx = 1 + \cos^{-1} x$

REŠITVE VAJ IZ KOTNIH FUNKCIJ

1. $\frac{\sqrt{2+\sqrt{3}}}{2}$, $\frac{\sqrt{2}-\sqrt{6}}{4}$	9. $2\sqrt{2}\sin\frac{x}{2}\cos(\frac{x}{2}-\frac{\pi}{4})$
2. 0	10. $\sin 2x - \frac{1}{2}\sin 4x$
3. $-\frac{1}{10}(3\sqrt{3}+4)$	11. $2\sin(x-\frac{\pi}{6})$
4. $\frac{\sqrt{2}}{6}(2-\sqrt{5})$	12. $-\frac{1}{2}\cos 2x + \frac{3}{2}$; $\omega = \pi$
5. $\frac{\pi}{4}$	13. $tg\alpha = \frac{\sqrt{2}}{2}$, $\sin\alpha = \pm\frac{\sqrt{3}}{3}$, $\cos\alpha = \pm\frac{\sqrt{6}}{3}$
6. $\frac{943}{1105}$	14. $2+\frac{\sqrt{3}}{3}$
7. 2	15. $tg2\alpha = -\frac{24}{7}$, $\cos 2\alpha = \pm\frac{7}{25}$
8. $4\cos\frac{\alpha+45^\circ}{2}\cos\frac{45^\circ-\alpha}{2}$	16. $arctg2$, $\lvert 10-4\pi\rvert$, $75-14\pi$, $-\frac{\pi}{4}$
	17. Rešitve enačb preveri s preizkusom.

List3

1. Izračunaj:

a) $\frac{\sin 120°}{\cos 135°}$ b) $\frac{1+\sin 315°}{\sin(-210°)}$ c) $\frac{\sin\frac{\pi}{3}}{\sin\left(-\frac{\pi}{6}\right)}$ d) $\frac{1-\cos\frac{3\pi}{4}}{1+\sin\frac{5\pi}{3}}$

e) $\frac{\sin 1050°-\sin 780°}{\cos^2(-840°)\cdot\cos 960°}$ f) $\frac{\cos 480°-\sin 540°-\sin(-330°)}{\cos(-540°)-\sin^2 750°}$ g) $\frac{\cos\frac{13\pi}{3}-\sin\left(-\frac{35\pi}{4}\right)}{\cos\frac{7\pi}{2}-\cos^2\frac{29\pi}{6}}$

2. Izračunaj $\sin\alpha$, če je $\cos\alpha=-\frac{2}{5}$ in je $\pi<\alpha<\frac{3\pi}{2}$.

3. Izračunaj $\frac{2-\cos\alpha}{\cos^2\alpha}$, če je $\sin\alpha=-\frac{1}{3}$ in je $\frac{3\pi}{2}<\alpha<2\pi$.

4. Poenostavi $\sin(x+360°)+\sin(720°-x)$.

5. Izračunaj $\sin\alpha\cdot\cos\alpha-1$, če je $\alpha=-\frac{17\pi}{3}$.

6. Poenostavi s pomočjo adicijskih izrekov:

a) $\sin(x+30°)+\sin(x-30°)$ b) $\cos(x-120°)-\cos(x+120°)$

c) $\frac{\sin(x+45°)-\sin(x-45°)}{\sin 45°}$ d) $\frac{\cos(x-120°)+\cos(x+120°)}{\sin(x-120°)-\sin(x+120°)}$

7. Izračunaj $\sin(x+150°)$, če je $\sin x=-\frac{2}{3}$ in je $180°<x<270°$.

8. Izračunaj $\cos(x-30°)$, če je $\cos x=\frac{5}{13}$ in je $270<x<360°$.

9. Izračunaj $\sin(\alpha-\beta)$, če je $\sin\alpha=\frac{3}{5}$ in $\cos\beta=-0.8$ ter $90°<\alpha,\beta<180°$.

10. Izračunaj $\sin 2x$ in $\cos 2x$, če je $\sin x=-\frac{4}{5}$ in je $270°<x<360°$.

11. Poenostavi $\cos\left(\frac{2\pi}{3}-x\right)+\cos\left(\frac{2\pi}{3}+x\right)+\cos x$.

12. Izračunaj $\sin(90°-x)+\cos(90°-x)-\sin(x+180°)+\cos(x+180°)$.

13. Poenostavi izraze:

a) $\frac{1+\cos 2x}{\sin 2x}$ b) $\frac{\sin 2x}{\cos 2x-1}$ c) $\frac{\sin 2x}{\cos 2x+\sin^2 x}$ d) $\frac{\sin x-\sin^3 x}{\sin 2x}$

e) $\frac{\cos x-\cos^3 x}{\sin 2x}$ f) $\frac{\sin^{-1} x-\sin x}{\cos^2 x}$ g) $\frac{\cos^{-1} x-\cos x}{\sin^2 x}-\cos x$

h) $\frac{\cos x-\sin x}{\cos x}\cdot\frac{\cos x+\sin x}{\sin x}$ i) $\frac{2\sin^2 x+\sin 2x}{2\cos^2 x+\sin 2x}$

14. Nariši grafe funkcij:

a) $f(x)=\sin x$ b) $f(x)=\cos x+1$ c) $f(x)=3\sin x$ d) $f(x)=-2\cos x$

e) $f(x)=\sin 2x$ f) $f(x)=\frac{1}{2}\cos 2x$ g) $f(x)=2\sin\frac{x}{2}-1$ h) $f(x)=-\cos 3x+1$

15. Izračunaj:

a) $tg135°-ctg315°$ b) $\frac{tg210°-tg(-135°)}{tg315°}$ c) $\frac{tg(-1050°)+ctg405°}{tg960°}$

16. Izračunaj $\sin x,\ tgx,\ ctgx$, če je $\cos x=-0.6$ in $90°<x<180°$.

17. Izračunaj $tg2x$, če je $\sin x=\frac{1}{4}$ in $90°<x<180°$.

18. Izračunaj $tg\left(x-\frac{2\pi}{3}\right)$, če je $\cos x=\frac{2}{3}$ in $\frac{3\pi}{2}<x<2\pi$.

19. Ugotovi, ali je dana funkcija soda ali liha:

a) $f(x)=x^2-x\sin x$ b) $f(x)=\sin x+x^3\cos x+tgx$ c) $f(x)=x-x^3\sin x-tgx$

20. Poenostavi:

a) $\frac{tgx}{1+ctg^2x}-\frac{1}{2}\sin 2x+tgx$ b) $\frac{tgx}{\sin^2 x}-\frac{ctgx}{\sin^{-2} x-1}$ c) $\frac{1}{\left(1+tg^2x\right)\sin x}+\frac{\cos x}{ctgx}$

d) $\frac{\sin^{-1} x-\sin x}{ctgx}+\frac{\left(1+tg^2x\right)\cos x}{1+ctg^2x}$ e) $\frac{\left(\cos^{-1} x-\cos x\right)ctgx}{1+ctg^2x}-tgx\cdot\cos x$

f) $\frac{\left(1+ctg^2x\right)\sin x}{1+tg^2x}+\frac{\cos^{-1}x-\cos x}{tgx}$

21.Izračunaj:

a) $\arcsin\frac{1}{2}$ b) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ c) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ d) $\arccos\frac{\sqrt{2}}{2}$

e) $\arccos\left(-\frac{1}{2}\right)$ f) $\arccos\frac{\sqrt{3}}{2}$ g) $\arccos(-1)$ h) $arctg1$

22.Reši enačbe:

a) $\sin x=\frac{1}{2}$ b) $\sin x=-\frac{\sqrt{2}}{2}$ c) $\sin x=-\frac{\sqrt{3}}{2}$ d) $\cos x=\frac{\sqrt{2}}{2}$

e) $\cos x=-\frac{1}{2}$ f) $\cos x=-1$ g) $tgx=\frac{\sqrt{3}}{3}$ h) $tgx=2$

23.Reši enačbe:

a) $\sin\frac{x}{2}=\frac{\sqrt{3}}{2}$ b) $\sin\left(x-\frac{\pi}{3}\right)=-\frac{1}{2}$ c) $\sin\left(3x-\frac{\pi}{4}\right)=-1$

d) $\cos 4x=-\frac{\sqrt{2}}{2}$ e) $2\cos\left(2x-\frac{\pi}{3}\right)=\sqrt{3}$ f) $\cos\left(x+\frac{2\pi}{6}\right)=1$

g) $tg\left(x+\frac{3\pi}{2}\right)=1$ h) $tg\left(2x-\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{3}$

24.Reši enačbe:

a) $6\sin^2x+13\sin x+5=0$ b) $2\cos^2x+\cos x-1=0$

c) $8\cos^2x-6\sin x=3$ d) $8\sin^2x-6\cos x-3=0$

e) $\sin 2x=\cos x$ f) $2\sin x=\sin 2x$

g) $ctgx\cdot\sin x=\cos^2x$ h) $2\sin^2x-5\sin x\cos x+3\cos^2x=0$

i) $7\sin^2x+7\sin x\cos x-2\cos^2x=1$ j) $7\sin^2x-8\sin x\cos x-2\cos^2x=2$

25.Izračunaj ničle in začetno vrednost funkcije $f(x)=2\sin\left(x-\frac{\pi}{4}\right)+1$.

26.Izračunaj točke, v katerih doseže funkcija $f(x)=2\sin\left(2x-\frac{\pi}{2}\right)$ maksimalno vrednost.

Rešitve:

1. a) $-\frac{\sqrt{6}}{2}$, b) $2-\sqrt{2}$, c) $-\sqrt{3}$, d) $\frac{2+\sqrt{2}}{2-\sqrt{3}}$, e) $4\left(1+\sqrt{3}\right)$, f) $\frac{4}{5}$, g) $-\frac{2+2\sqrt{2}}{3}$,

2. $-\frac{\sqrt{21}}{5}$,

3. $\frac{9-3\sqrt{2}}{4}$,

4. 0,

5. $\frac{\sqrt{3}-4}{4}$,

6. a) $\sqrt{3}\sin x$, b) $\sqrt{3}\sin x$, c) $2\cos x$, d) $\frac{\sqrt{3}}{3}$,

7. $\cos x=-\frac{\sqrt{5}}{3}$, $\sin(x+150°)=\frac{2\sqrt{3}-\sqrt{5}}{6}$,

8. $\sin x=-\frac{12}{13}$, $\cos(x-30°)=\frac{5\sqrt{3}-12}{26}$,

9. $\cos\alpha=-\frac{4}{5}$, $\sin\beta=\frac{3}{5}$, $\sin(\alpha-\beta)=0$,

10. $\cos x=\frac{3}{5}$, $\sin 2x=-\frac{24}{25}$, $\cos 2x=-\frac{7}{25}$,

11. 0,

12. $2\sin x$,

13. a) ctg$x$, b) - ctg$x$, c) 2tg$x$, d) $\frac{\cos x}{2}$, e) $\frac{\sin x}{2}$, f) $\frac{1}{\sin x}$, g) $\frac{\sin^2x}{\cos x}$, h) $2ctg2x$, i) tg$x$,

15. a) 0, b) $\frac{3-\sqrt{3}}{3}$, c) $\frac{1+\sqrt{3}}{3}$,

16. $\sin x=\frac{4}{5}$, $tgx=-\frac{4}{3}$, $ctgx=-\frac{3}{4}$,

17. $\cos x=-\frac{\sqrt{15}}{4}$, $tgx=-\frac{\sqrt{15}}{15}$, $tg2x=-\frac{\sqrt{15}}{7}$,

18. $\sin x=-\frac{\sqrt{5}}{3}$, $tgx=-\frac{\sqrt{5}}{2}$, $tg\left(x-\frac{2\pi}{3}\right)=\frac{2\sqrt{3}-\sqrt{5}}{2+\sqrt{15}}$,

19. a) soda, b) liha, c) niti soda niti liha,

20. a) $\frac{2\sin^3 x}{\cos x}$, b) ctg$x$, c) $\frac{1}{\sin x}$, d) $\frac{1}{\cos x}$, e) $-\sin x\cos^2 x$, f) $\frac{1}{\sin x}$,

21. a) $\frac{\pi}{6}$, b) $-\frac{\pi}{3}$, c) $-\frac{\pi}{4}$, d) $\frac{\pi}{4}$, e) $\frac{2\pi}{3}$, f) $\frac{\pi}{6}$, g) π, h) $\frac{\pi}{4}$,

22. a) $x_1 = \frac{\pi}{6} + 2\pi k,\ x_2 = \frac{5\pi}{6} + 2\pi k,\ k \in Z$, b) $x_1 = -\frac{\pi}{4} + 2\pi k,\ x_2 = \frac{5\pi}{4} + 2\pi k,\ k \in Z$,

c) $x_1 = -\frac{\pi}{3} + 2\pi k,\ x_2 = \frac{4\pi}{3} + 2\pi k,\ k \in Z$, d) $x_1 = \frac{\pi}{4} + 2\pi k,\ x_2 = -\frac{\pi}{4} + 2\pi k,\ k \in Z$,

e) $x_1 = \frac{2\pi}{3} + 2\pi k,\ x_2 = -\frac{2\pi}{3} + 2\pi k,\ k \in Z$, f) $x = \pi + 2\pi k,\ k \in Z$, g) $x = -\frac{\pi}{6} + \pi k,\ k \in Z$,

h) $x = arctg2 + \pi k,\ k \in Z$,

23. a) $x_1 = \frac{2\pi}{3} + 4\pi k,\ x_2 = \frac{4\pi}{3} + 4\pi k,\ k \in Z$, b) $x_1 = \frac{\pi}{6} + 2\pi k,\ x_2 = \frac{3\pi}{2} + 2\pi k,\ k \in Z$,

c) $x = -\frac{\pi}{12} + \frac{2\pi k}{3},\ k \in Z$, d) $x_1 = \frac{3\pi}{16} + \frac{\pi k}{2},\ x_2 = -\frac{3\pi}{16} + \frac{\pi k}{2},\ k \in Z$, e) $x_1 = \frac{\pi}{4} + \pi k,\ x_2 = \frac{\pi}{12} + \pi k,\ k \in Z$,

f) $x = -\frac{\pi}{3} + 2\pi k,\ k \in Z$, g) $x = -\frac{5\pi}{4} + \pi k,\ k \in Z$, h) $x = \frac{\pi}{24} + \frac{\pi k}{2},\ k \in Z$,

24. a) $x_1 = -\frac{\pi}{6} + 2\pi k,\ x_2 = \frac{7\pi}{6} + 2\pi k,\ k \in Z$,

b) $x_1 = \pi + 2\pi k,\ x_2 = \frac{\pi}{3} + 2\pi k,\ x_3 = -\frac{\pi}{3} + 2\pi k,\ k \in Z$, c) $x_1 = \frac{\pi}{6} + 2\pi k,\ x_2 = \frac{5\pi}{6} + 2\pi k,\ k \in Z$,

d) $x_1 = \frac{\pi}{6} + 2\pi k,\ x_2 = -\frac{\pi}{6} + 2\pi k,\ k \in Z$, e) $x_1 = \frac{\pi}{2} + \pi k,\ x_2 = \frac{\pi}{6} + 2\pi k,\ x_3 = \frac{5\pi}{6} + 2\pi k,\ k \in Z$, f) $x = 2\pi k,\ k \in Z$,

g) $x_1 = \frac{\pi}{2} + \pi k,\ x_2 = 2\pi k,\ k \in Z$, h) $x_1 = arctg\frac{3}{2} + \pi k,\ x_2 = \frac{\pi}{4} + \pi k,\ k \in Z$,

i) $x_1 = arctg\left(-\frac{3}{2}\right) + \pi k,\ x_2 = arctg\frac{1}{3} + \pi k,\ k \in Z$, j) $x_1 = arctg2 + \pi k,\ x_2 = arctg\left(-\frac{2}{5}\right) + \pi k,\ k \in Z$,

25. ničle: $x_1 = \frac{\pi}{12} + 2\pi k,\ x_2 = \frac{17\pi}{12} + 2\pi k,\ k \in Z$, zač. vrednost: $f(0) = -\sqrt{2+1}$,

26. $x = \frac{\pi}{2} + \pi k,\ k \in Z$